1.- Racionaliza:

a) $\frac{5+\sqrt{3}}{5-\sqrt{3}}=\frac{\left(5+\sqrt{3}\right)^2}{\left(5-\sqrt{3}\right)\cdot\left(5+\sqrt{3}\right)}=\frac{25+3+10\sqrt{3}}{22}=\frac{14+5\sqrt{3}}{11}$

b) $\frac{1}{\sqrt{2-\sqrt{3}}}=\frac{\sqrt{2-\sqrt{3}}}{2-\sqrt{3}}=\frac{\left(2+\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{4-3}=2\sqrt{2-\sqrt{3}}+\sqrt{6-3\sqrt{3}}$

c) $\frac{x}{\sqrt[3]{1-\sqrt{x}}}=\frac{x\sqrt[3]{\left(1-\sqrt{x}\right)^2}}{1-\sqrt{x}}=\frac{\left(x+x\sqrt{x}\right)\sqrt[3]{\left(1-\sqrt{x}\right)^2}}{1-x}$

2.- Efectúa las siguientes operaciones, simplificando el resultado:

a) $\frac{5-\sqrt{3}}{5+\sqrt{3}}-\frac{5+\sqrt{3}}{5-\sqrt{3}}=\frac{\left(5-\sqrt{3}\right)^2-\left(5+\sqrt{3}\right)^2}{5-3}=\frac{-20\sqrt{3}}{2}=-10\sqrt{3}$

b) $\frac{1}{\left(1-\sqrt{x}\right)^2}=\frac{1}{1+x-2\sqrt{x}}=\frac{1+x+2\sqrt{x}}{(1+x)^2-4x}=\frac{1+x+2\sqrt{x}}{1+x^2-2x}$

c) $\frac{\sqrt{3}-\sqrt{8}}{\sqrt{3}+\sqrt{8}}-\frac{24}{5\sqrt{6}}=\frac{\left(\sqrt{3}-\sqrt{8}\right)^2}{3-8}-\frac{24\sqrt{6}}{5\cdot 6}=\frac{3+8-2\sqrt{24}}{-5}-\frac{4\sqrt{6}}{5}=-\frac{11}{5}$

3.- Racionaliza:

a) $\frac{1}{1+\frac{\sqrt{3}}{\sqrt{2}-\sqrt{3}}}=\frac{1}{\frac{\sqrt{2}}{\sqrt{2}-\sqrt{3}}}=\frac{1}{\frac{2+\sqrt{6}}{-1}}=\frac{-1}{2+\sqrt{6}}=\frac{\sqrt{6}-2}{-4}=\frac{2-\sqrt{6}}{4}$

b) $\frac{1}{\sqrt{1+x-\sqrt{x}}}=\frac{\sqrt{1+x-\sqrt{x}}}{1+x-\sqrt{x}}=\frac{\left(1+x+\sqrt{x}\right)\sqrt{1+x-\sqrt{x}}}{(1+x)^2-x}=\frac{\left(1+x+\sqrt{x}\right)\sqrt{1+x-\sqrt{x}}}{1+x+x^2}$

c) $\frac{3\sqrt{75}}{4\sqrt{18}}=\frac{3\cdot 5\sqrt{3}}{4\cdot 3\sqrt{2}}=\frac{5\sqrt{6}}{8}$

d) $\frac{5\sqrt{5}}{\sqrt{25}-\sqrt{20}}=\frac{5\sqrt{5}}{5-2\sqrt{5}}=\frac{5\sqrt{5}\cdot\left(5+2\sqrt{5}\right)}{25-20}=\frac{5\cdot\left(5\sqrt{5}+2\cdot 5\right)}{5}=5\sqrt{5}+10$

e) $\frac{2\sqrt{2}}{1-\sqrt{2x^2}}=\frac{2\sqrt{2}\cdot\left(1+x\sqrt{2}\right)}{\left(1-x\sqrt{2}\right)\cdot\left(1+x\sqrt{2}\right)}=\frac{2\sqrt{2}-2x}{1-2x^2}$

f) $\frac{3\sqrt{3}}{\sqrt{27}}=\frac{3\sqrt{3}}{3\sqrt{3}}=1$

4.- Efectúa las siguientes operaciones, simplificando el resultado:

a) $\frac{4\sqrt{6}\cdot 2\sqrt{3}}{5\sqrt{8}\cdot 3\sqrt{12}}=\frac{4\sqrt{6}\cdot 2\sqrt{3}}{10\sqrt{2}\cdot 6\sqrt{3}}=\frac{2\sqrt{3}}{15}$

b) $\frac{a\sqrt{b^2ac^3}\cdot\sqrt[3]{abc^5}}{c\sqrt{ab^2c}\cdot\sqrt[3]{a^3b^5c}}=\frac{abc\sqrt{ac}}{cb\sqrt{ac}}\cdot\frac{c\sqrt[3]{abc^2}}{ab\sqrt[3]{b^2c}}=\frac{c}{b}\cdot\frac{\sqrt[3]{abc^2}\cdot\sqrt[3]{bc^2}}{bc}=\frac{c\sqrt[3]{ab^2c}}{b^2}$

5.- Efectúa las siguientes operaciones, racionalizando previamente las fracciones:

a) $\frac{3}{\sqrt{15}}+\frac{\sqrt{2}}{3\sqrt{5}}=\frac{3\sqrt{15}}{15}+\frac{\sqrt{10}}{15}=\frac{3\sqrt{15}+\sqrt{10}}{15}$

b)

$$\frac{2+\sqrt{6}}{3\sqrt{6}-\sqrt{2}}=\frac{\left(2+\sqrt{6}\right)\cdot\left(3\sqrt{6}-\sqrt{2}\right)}{54-2}=\frac{6\sqrt{6}-2\sqrt{2}+18-2\sqrt{3}}{52}=\frac{3\sqrt{6}-\sqrt{2}+9-\sqrt{3}}{26}$$

$$\frac{20}{6\sqrt{7}-10\sqrt{2}}=\frac{120\sqrt{7}+200\sqrt{2}}{252-200}=\frac{60\sqrt{7}-100\sqrt{2}}{26}$$

➔ $$\frac{2+\sqrt{6}}{3\sqrt{6}-\sqrt{2}}-\frac{20}{6\sqrt{7}-10\sqrt{2}}=\frac{3\sqrt{6}-99\sqrt{2}-\sqrt{3}-60\sqrt{7}+9}{26}$$

6.- Efectúa las siguientes operaciones, simplificando el resultado:

a) $\frac{3}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt[3]{3}}=\frac{3\sqrt{3}}{3}-\frac{\sqrt{3}\cdot\sqrt[3]{3^2}}{3}=\frac{\left(3-\sqrt[3]{3^2}\right)\sqrt{3}}{3}$

b) $\left(-2\sqrt{5}\right)^2=4\cdot 5=20$

c) $\frac{\left(\sqrt{3}+2\right)\cdot\left(2-\sqrt{3}\right)}{1-\sqrt{3}}=\frac{\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)}{1-\sqrt{3}}=\frac{(4-3)\cdot\left(1+\sqrt{3}\right)}{-2}=-\frac{\left(1+\sqrt{3}\right)}{2}$

d) $\frac{3\sqrt{20}}{5\sqrt{5}}-\frac{1}{\sqrt{5}}=\frac{3\sqrt{100}}{25}-\frac{\sqrt{5}}{5}=\frac{6}{5}-\frac{\sqrt{5}}{5}=\frac{6-\sqrt{5}}{5}$

e) $\frac{2\sqrt{2}\cdot 3\sqrt{3}}{\sqrt{24}}=\frac{6\sqrt{6}}{2\sqrt{6}}=3$

f) $\frac{\left(3\sqrt{5}+5\sqrt{3}\right)\cdot\left(5\sqrt{5}-3\sqrt{3}\right)}{\sqrt{5}+\sqrt{3}}=\frac{15\cdot 5+25\sqrt{15}-9\sqrt{15}-15\cdot 3}{\sqrt{5}+\sqrt{3}}=\frac{30+16\sqrt{15}}{\sqrt{5}+\sqrt{3}}=$

$$=\frac{\left(30+16\sqrt{5}\right)\cdot\left(\sqrt{5}-\sqrt{3}\right)}{2}=\left(15+8\sqrt{5}\right)\cdot\left(\sqrt{5}-\sqrt{3}\right)=15\sqrt{5}+40-15\sqrt{3}-8\sqrt{15}$$