

Ecuaciones trigonométricas

7.- $\operatorname{tg} x \cdot \operatorname{sec} x = \sqrt{2}$

$$\frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2} ; \quad \frac{\operatorname{sen} x}{\cos^2 x} = \sqrt{2} ; \quad \operatorname{sen} x = \sqrt{2}(1 - \operatorname{sen}^2 x) ; \quad \sqrt{2}\operatorname{sen}^2 x + \operatorname{sen} x - \sqrt{2} = 0$$

$$\operatorname{sen} x = \begin{cases} \frac{\sqrt{2}}{2} \\ -\sqrt{2} \end{cases} \quad (\text{la solución } \operatorname{sen} x = -\sqrt{2} \text{ no es válida})$$

$$\operatorname{sen} x = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 135^\circ + 360^\circ \cdot k \end{cases}$$

8.- $\operatorname{tg} 2x = -\operatorname{tg} x$

$$\frac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x} = -\operatorname{tg} x ; \quad 2\operatorname{tg} x = -\operatorname{tg} x + \operatorname{tg}^3 x ; \quad 3\operatorname{tg} x - \operatorname{tg}^3 x = 0 ; \quad \operatorname{tg} x(3 - \operatorname{tg}^2 x) = 0 ;$$

$\operatorname{tg} x = 0 \rightarrow x_1 = 0^\circ + 180^\circ \cdot k$ (el valor de la tangente se repite cada 180°)

$$3 - \operatorname{tg}^2 x = 0 ; \quad \operatorname{tg}^2 x = 3 ; \quad \operatorname{tg} x = \pm \sqrt{3} ;$$

$$\operatorname{tg} x = +\sqrt{3} \rightarrow x_2 = 60^\circ + 180^\circ \cdot k$$

$$\operatorname{tg} x = -\sqrt{3} \rightarrow x_3 = 120^\circ + 180^\circ \cdot k$$

9.- $\cos 2x + \operatorname{sen} x = \operatorname{sen} 3x$

$$\cos 2x = \operatorname{sen} 3x - \operatorname{sen} x ; \quad (\text{Recuerda: } \operatorname{sen} A - \operatorname{sen} B = 2 \cos \frac{A+B}{2} \operatorname{sen} \frac{A-B}{2})$$

$$\cos 2x = 2 \cos \frac{3x+x}{2} \operatorname{sen} \frac{3x-x}{2} ; \quad \cos 2x = 2 \cos 2x \operatorname{sen} x ;$$

$$\cos 2x - 2 \cos 2x \operatorname{sen} x = 0 ; \quad \cos 2x(1 - 2 \operatorname{sen} x) = 0 ;$$

$$\cos 2x = 0 \rightarrow 2x = 90^\circ + 180^\circ \cdot k \rightarrow x_1 = 45^\circ + 90^\circ \cdot k$$

$$1 - 2\operatorname{sen} x = 0 ; \quad 2\operatorname{sen} x = 1 ; \quad \operatorname{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x_2 = 30^\circ + 360^\circ \cdot k \\ x_3 = 150^\circ + 360^\circ \cdot k \end{cases}$$

10.- $\cos^2 x - 3 \operatorname{sen}^2 x = 0$

$$1 - \operatorname{sen}^2 x - 3 \operatorname{sen}^2 x = 0 ; \quad 1 - 4 \operatorname{sen}^2 x = 0 ; \quad \operatorname{sen}^2 x = \frac{1}{4} ; \quad \operatorname{sen} x = \sqrt{\frac{1}{4}} = \pm \frac{1}{2} ;$$

$$\operatorname{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases} \quad \operatorname{sen} x = -\frac{1}{2} \Rightarrow \begin{cases} x_3 = 210^\circ + 360^\circ \cdot k \\ x_4 = 330^\circ + 360^\circ \cdot k \end{cases}$$

11.- $\operatorname{sen}^2 x - \cos^2 x = \frac{1}{2}$

$$1 - \cos^2 x - \cos^2 x = \frac{1}{2} ; \quad 1 - 2 \cos^2 x = \frac{1}{2} ; \quad 2 - 4 \cos^2 x = 1 ; \quad \cos^2 x = \frac{1}{4} ; \quad \cos x = \pm \frac{1}{2}$$

$$\cos x = \frac{1}{2} \Rightarrow \begin{cases} x_1 = 60^\circ + 360^\circ \cdot k \\ x_2 = 300^\circ + 360^\circ \cdot k \end{cases} \quad \cos x = -\frac{1}{2} \Rightarrow \begin{cases} x_3 = 120^\circ + 360^\circ \cdot k \\ x_4 = 240^\circ + 360^\circ \cdot k \end{cases}$$