

Ejercicios resueltos

39.-
$$\frac{\operatorname{tg} \alpha - \operatorname{cot} \alpha}{\operatorname{tg} \alpha + \operatorname{cot} \alpha} = \operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha$$

$$\begin{aligned}\frac{\operatorname{tg} \alpha - \operatorname{cot} \alpha}{\operatorname{tg} \alpha + \operatorname{cot} \alpha} &= \frac{\frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} - \frac{\operatorname{cos} \alpha}{\operatorname{sen} \alpha}}{\frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} + \frac{\operatorname{cos} \alpha}{\operatorname{sen} \alpha}} = \frac{\frac{\operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha}{\operatorname{cos} \alpha \operatorname{sen} \alpha}}{\frac{\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha}{\operatorname{cos} \alpha \operatorname{sen} \alpha}} = \frac{\operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha}{\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha} = \operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha\end{aligned}$$

40.-
$$\frac{\sec a - \operatorname{cos} a}{\sec a + \operatorname{cos} a} = \frac{\operatorname{sen}^2 a}{1 + \operatorname{cos}^2 a}$$

$$\begin{aligned}\frac{\sec a - \operatorname{cos} a}{\sec a + \operatorname{cos} a} &= \frac{\frac{1}{\operatorname{cos} a} - \operatorname{cos} a}{\frac{1}{\operatorname{cos} a} + \operatorname{cos} a} = \frac{\frac{1 - \operatorname{cos}^2 a}{\operatorname{cos} a}}{\frac{1 + \operatorname{cos}^2 a}{\operatorname{cos} a}} = \frac{1 - \operatorname{cos}^2 a}{1 + \operatorname{cos}^2 a} = \frac{\operatorname{sen}^2 a}{1 + \operatorname{cos}^2 a}\end{aligned}$$

41.-
$$\frac{\sec a}{1 + \sec a} = \frac{1 - \operatorname{cos} a}{\operatorname{sen}^2 a}$$

$$\begin{aligned}\frac{\sec a}{1 + \sec a} &= \frac{\frac{1}{\operatorname{cos} a}}{1 + \frac{1}{1 + \frac{1}{\operatorname{cos} a}}} = \frac{\frac{1}{\operatorname{cos} a}}{\frac{\operatorname{cos} a + 1}{\operatorname{cos} a}} = \frac{1}{\operatorname{cos} a + 1} = \frac{1 - \operatorname{cos} a}{(1 + \operatorname{cos} a)(1 - \operatorname{cos} a)} = \frac{1 - \operatorname{cos} a}{1 - \operatorname{cos}^2 a} = \frac{1 - \operatorname{cos} a}{\operatorname{sen}^2 a}\end{aligned}$$

42.-
$$\frac{\operatorname{tg} \alpha - \operatorname{cot} \alpha}{\operatorname{tg} \alpha + \operatorname{cot} \alpha} = 2 \operatorname{sen}^2 \alpha - 1$$

$$\begin{aligned}\frac{\operatorname{tg} \alpha - \operatorname{cot} \alpha}{\operatorname{tg} \alpha + \operatorname{cot} \alpha} &= \frac{\frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} - \frac{\operatorname{cos} \alpha}{\operatorname{sen} \alpha}}{\frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} + \frac{\operatorname{cos} \alpha}{\operatorname{sen} \alpha}} = \frac{\frac{\operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha}{\operatorname{cos} \alpha \operatorname{sen} \alpha}}{\frac{\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha}{\operatorname{cos} \alpha \operatorname{sen} \alpha}} = \frac{\operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha}{\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha} = \operatorname{sen}^2 \alpha - \operatorname{cos}^2 \alpha = \\ &= \operatorname{sen}^2 \alpha - (1 - \operatorname{sen}^2 \alpha) = \operatorname{sen}^2 \alpha - 1 + \operatorname{sen}^2 \alpha = 2 \operatorname{sen}^2 \alpha - 1\end{aligned}$$

43.-
$$\frac{\sec a + \operatorname{tg} \alpha}{\operatorname{cot} a + \operatorname{cos} \alpha} = \operatorname{tg} \alpha \operatorname{sec} \alpha$$

$$\begin{aligned}\frac{\sec a + \operatorname{tg} \alpha}{\operatorname{cot} a + \operatorname{cos} \alpha} &= \frac{\frac{1}{\operatorname{cos} \alpha} + \frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha}}{\frac{\operatorname{cos} \alpha}{\operatorname{sen} \alpha} + \operatorname{cos} \alpha} = \frac{\frac{1 + \operatorname{sen} \alpha}{\operatorname{cos} \alpha}}{\frac{\operatorname{cos} \alpha + \operatorname{sen} \alpha \operatorname{cos} \alpha}{\operatorname{sen} \alpha}} = \frac{(1 + \operatorname{sen} \alpha) \operatorname{sen} \alpha}{\operatorname{cos} \alpha (\operatorname{cos} \alpha + \operatorname{sen} \alpha \operatorname{cos} \alpha)} = \\ &= \frac{(1 + \operatorname{sen} \alpha) \operatorname{sen} \alpha}{\operatorname{cos}^2 \alpha (1 + \operatorname{sen} \alpha)} = \frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} \cdot \frac{1}{\operatorname{cos} \alpha} = \operatorname{tg} \alpha \operatorname{sec} \alpha\end{aligned}$$