Ejercicios resueltos

39.- $\dfrac{tg\,\alpha-\cot\alpha}{tg\,\alpha+\cot\alpha}=sen^2\alpha-\cos^2\alpha$

$$\frac{tg\,\alpha-\cot\alpha}{tg\,\alpha+\cot\alpha}=\frac{\dfrac{sen\,\alpha}{\cos\alpha}-\dfrac{\cos\alpha}{sen\,\alpha}}{\dfrac{sen\,\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{sen\,\alpha}}=\frac{\dfrac{sen^2\alpha-\cos^2\alpha}{\cos\alpha\,sen\,\alpha}}{\dfrac{sen^2\alpha+\cos^2\alpha}{\cos\alpha\,sen\,\alpha}}=\frac{sen^2\alpha-\cos^2\alpha}{sen^2\alpha+\cos^2\alpha}=sen^2\alpha-\cos^2\alpha$$

40.- $\dfrac{\sec a-\cos\alpha}{\sec a+\cos\alpha}=\dfrac{sen^2\,a}{1+\cos^2 a}$

$$\frac{\sec a-\cos\alpha}{\sec a+\cos\alpha}=\frac{\dfrac{1}{\cos\alpha}-\cos\alpha}{\dfrac{1}{\cos\alpha}+\cos\alpha}=\frac{\dfrac{1-\cos^2 a}{\cos\alpha}}{\dfrac{1+\cos^2 a}{\cos\alpha}}=\frac{1-\cos^2 a}{1+\cos^2 a}=\frac{sen^2\,a}{1+\cos^2 a}$$

41.- $\dfrac{\sec a}{1+\sec a}=\dfrac{1-\cos a}{sen^2\,a}$

$$\frac{\sec a}{1+\sec a}=\frac{\dfrac{1}{\cos a}}{1+\dfrac{1}{\cos a}}=\frac{\dfrac{1}{\cos a}}{\dfrac{\cos a+1}{\cos a}}=\frac{1}{\cos a+1}=\frac{1-\cos a}{(1+\cos a)(1-\cos a)}=\frac{1-\cos a}{1-\cos^2 a}=\frac{1-\cos a}{sen^2\,a}$$

42.- $\dfrac{tg\,\alpha-\cot\alpha}{tg\,\alpha+\cot\alpha}=2\,sen^2\alpha-1$

$$\frac{tg\,\alpha-\cot\alpha}{tg\,\alpha+\cot\alpha}=\frac{\dfrac{sen\,\alpha}{\cos\alpha}-\dfrac{\cos\alpha}{sen\,\alpha}}{\dfrac{sen\,\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{sen\,\alpha}}=\frac{\dfrac{sen^2\alpha-\cos^2\alpha}{sen\,\alpha\cos\alpha}}{\dfrac{sen^2\alpha+\cos^2\alpha}{sen\,\alpha\cos\alpha}}=\frac{sen^2\alpha-\cos^2\alpha}{sen^2\alpha+\cos^2\alpha}=sen^2\alpha-\cos^2\alpha=$$

$$=sen^2\alpha-(1-sen^2\alpha)=sen^2\alpha-1+sen^2\alpha=2\,sen^2\alpha-1$$

43.- $\dfrac{\sec a+tg\,\alpha}{\cot a+\cos\alpha}=tg\,\alpha\sec\alpha$

$$\frac{\sec a+tg\,\alpha}{\cot a+\cos\alpha}=\frac{\dfrac{1}{\cos\alpha}+\dfrac{sen\,\alpha}{\cos\alpha}}{\dfrac{\cos\alpha}{sen\,\alpha}+\cos\alpha}=\frac{\dfrac{1+sen\,\alpha}{\cos\alpha}}{\dfrac{\cos\alpha+sen\,\alpha\cos\alpha}{sen\,\alpha}}=\frac{(1+sen\,\alpha)\,sen\,\alpha}{\cos\alpha(\cos\alpha+sen\,\alpha\cos\alpha)}=$$

$$=\frac{(1+sen\,\alpha)\,sen\,\alpha}{\cos^2\alpha(1+sen\,\alpha)}=\frac{sen\,\alpha}{\cos\alpha}\cdot\frac{1}{\cos\alpha}=tg\,\alpha\sec\alpha$$